The relationship between the height of water level in a glass and the frequency of the emitted sound

Introduction, aim and background information

On my last trip to Budapest, Hungary, I watched a street musician play an instrument called the glass xylophone. A glass xylophone is a musical instrument made of upright glasses that are filled with water to different heights. There are many ways to make a glass xylophone, simpler instruments consist of just a few glasses, while more complex ones are carefully designed and can include over fifteen glasses. Observing the street musician play the glass xylophone made me curious about the Physics behind this instrument, so for my internal assessment I decided to investigate how a glass xylophone works, more specifically, why different glasses produce different pitches depending on the height of the water in the glass. Pitch in music simply means how high or low a sound is ("Pitch"). When studying waves, I remember learning that higher pitch means that the sound wave has a higher frequency, while a lower pitch means lower frequency. Based on what I know so far, I decided on the following research question for my investigation:

How does the height of the water level in a glass affect the frequency of the sound emitted by the glass when the side of the glass is hit with a spoon?

When I started researching glass xylophones, I found many more articles about singing glasses, also called glass harps. In contrast to a glass xylophone, when playing a glass harp one has to rub the glass rim with a finger to produce sound (Nose 4). I also discovered that it does not matter whether we rub our wet fingers on the rim of the glass or hit the side of the glass with a metal spoon, because both methods create vibrations in the glass and produce the same frequency, so in my work I will also refer to articles about singing glasses ("Singing Glasses"). When energy is transferred to the glass by rubbing its rim or by hitting its side with a spoon, the glass starts to vibrate. In turn, this causes the air molecules to vibrate with the same frequency, producing the sound that we hear (Nose 10). As we add more water, it becomes more difficult for the glass wall to vibrate due to the added mass and as a result, vibrates at lower frequencies, producing a lower pitch (Lee 1). Moreover, I also found that the relationship between the height of the water level and the frequency of the sound is not linear (Lee 3, Nose 11). Hence my hypothesis is that as the water level in the glass increases, the frequency of the sound produced by the vibrating glass will decrease at an increasing rate, producing a nonlinear curve.

Planning

I. Variables

1. Independent variable: Height of the water level in the glass (cm)
2. Dependent variable: Frequency of the emitted sound wave (Hz)
3. Control variables

    i. Glass used

    - Reason: glass shape and size influence the vibrations produced by the glass wall, hence affect the frequency.

- Method: I will use the same glass throughout the experiment.

ii. Glass cross-sectional area
   - Reason: the volume of the water might influence the frequency. If the glass has varying cross-sectional area, every time we add water, the volume of the water added will be different.
   - Method: I will use a glass that has the same cross-sectional area along its entire height.

iii. Liquid used in the glasses
   - Reason: different liquids have different densities. Larger density makes it more difficult for the glass to vibrate, hence affects frequency.
   - Method: I will use water throughout the experiment.

iv. Air temperature
   - Reason: the speed of sound depends on the temperature of the medium it travels in. Since frequency is related to wave speed, temperature should be the same throughout the experiment
   - Method: I will carry out the experiment indoors so that temperature can be kept constant.

II. Apparatus

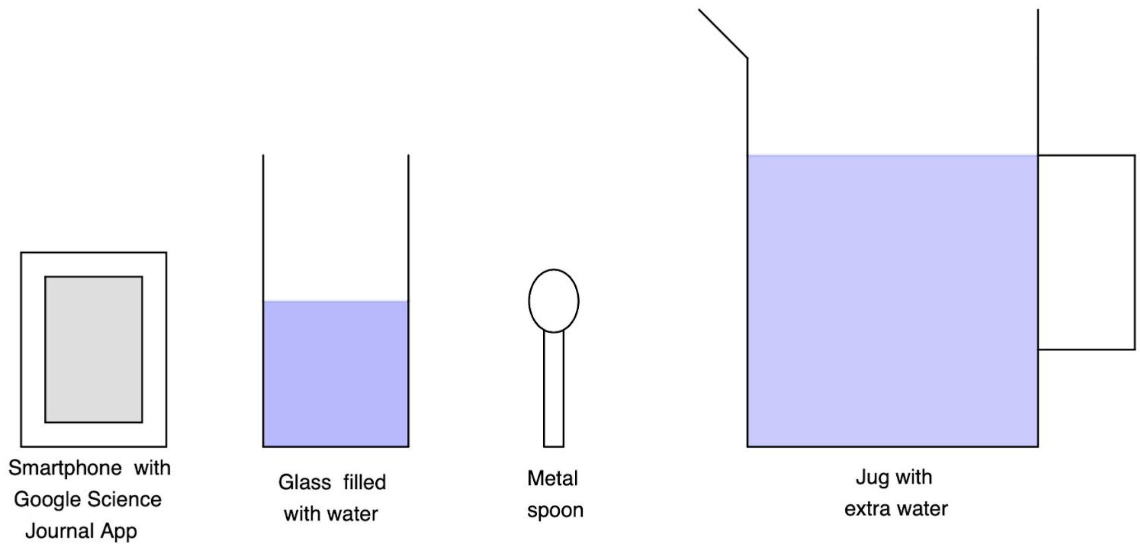


Figure 1. Setup of apparatus.

III. Method

1. Place smartphone approximately 10 cm from the empty glass so that its microphone is pointing towards the glass, open the Google Science Journal App and turn on the Pitch sensor
2. Start the Pitch sensor
3. Hit the side of the glass with the spoon
4. Stop the Pitch sensor
5. Record maximum frequency displayed on the screen
6. Repeat steps 2 to 5 four more times
7. Increase water level by 2.0 cm
8. Repeat steps 2 to 7 until the water level reaches 12.0 cm

IV. Safety, ethical and environmental considerations

In general, this is a safe experiment. Since I am working with water, I will make sure that when I am recording the frequencies, the smartphone is not connected to a charger. I will also take care with the glass and ther jar, since these can break and potentially cause injury. There are no animals or people involved in this experiment, so there are no major ethical considerations. In addition, the experiment does not harm the environment.

Data collection and analysis

I. Raw data

Uncertainties: in the table below, the uncertainty in *H* is ± 0.2 cm, because even though I used a ruler with a millimeter scale division to measure the height of the water level, it was somewhat difficult to read this measurement, so I chose a larger uncertainty to be on the safe side. The uncertainty in *f* is ± 0.1 Hz, because the Google Science Journal App shows the measured frequency to one decimal place.

Table 1. Raw data collected from the experiment.

Height *H* / cm $\Delta H = \pm 0.2$ cm		0	2.0	4.0	6.0	8.0	10.0	12.0
Frequency *f* / Hz $\Delta f = \pm 0.1$ Hz	Trial 1	1156.3	1149.1	1139.7	1115.7	1030.2	908.7	709.1
	Trial 2	1157.1	1148.3	1139.2	1116.0	1028.9	908.9	709.6
	Trial 3	1156.5	1148.9	1138.9	1115.7	1028.6	908.2	709.8
	Trial 4	1156.7	1148.7	1139.3	1115.4	1029.5	908.7	708.8
	Trial 5	1156.8	1148.9	1139.5	1115.1	1028.6	908.4	709.0

II. Data processing

1. Average frequency (f_{Ave})

   I will calculate the average frequency for each value of *H* by adding up the frequency values that I received for the five trials and dividing by five.

   Example calculation for *H* = 0:

   $$\frac{1156.3+1157.1+1156.5+1156.7+1156.8}{5} = 1156.68 \text{ Hz} \approx 1156.7 \text{ Hz}$$

2. Absolute uncertainty in the average frequency (Δf_{Ave})

   I will calculate the uncertainty in the average frequency for each value of *H* by subtracting the smallest value of *f* from the largest value of *f* for the given value of *H* and divide the result by two.

   Example calculation for *H* = 0:

   $$\frac{1157.1-1156.3}{2} = 0.4 \text{ Hz}$$

3. Percentage uncertainty in the average frequency ($\frac{\Delta f_{Ave}}{f_{Ave}} \times 100\ \%$)

   I will calculate the percentage uncertainty in the average frequency for each value of *H* by dividing the absolute uncertainty in the frequency by the average frequency and multiply the result by 100 %.

   Example calculation for *H* = 0:

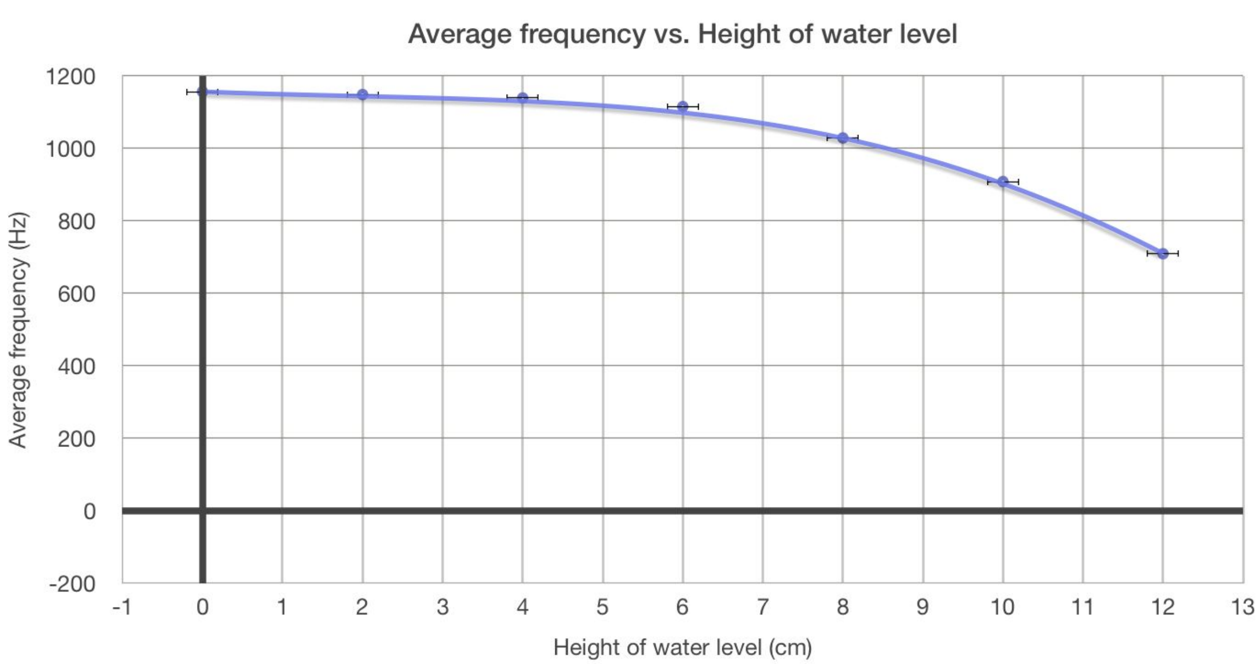
   $$\frac{0.4}{1156.7} \times 100\ \% = 0.03\ \%$$

Table 2. Processed data.

H (cm) ± 0.2 cm	f_{Ave} (Hz)	Δf_{Ave} (Hz)	$\frac{\Delta f_{Ave}}{f_{Ave}} \times 100\ \%$
0	1156.7	0.4	0.03 %
2.0	1148.8	0.4	0.03 %
4.0	1139.3	0.4	0.04 %
6.0	1115.6	0.5	0.04 %
8.0	1029.0	0.8	0.08 %
10.0	908.6	0.4	0.04 %
12.0	709.3	0.5	0.07%

III. Graphing the results

Graph 1. Average frequency vs. Height of water level



As explained earlier, the uncertainty in H is ± 0.2 cm. Horizontal error bars are displayed on the graph to show this.From the last column of Table 2 we can see that the percentage uncertainty in the average frequency is negligible, so I decided not to include vertical error bars.The graph clearly shows that as *H* increases, *f* decreases and that there is a nonlinear relationship between the variables. This is consistent with the hypothesis and the underlying concepts that were described in the introduction. Using the Trendline function of the program that I used to draw the graph, I tried to fit different models to the data set. After experimenting with various options, the best fit seemed to be a fourth order polynomial. I will return to this equation in the Conclusion section. The trendline shown on the diagram (Graph 1) is the graph of this polynomial. The R^2 value for this polynomial trendline is 0.9994. Since this value is very close to 1, it confirms that the trendline fits the data very well.

<u>Conclusion and evaluation</u>

I. Conclusion

1. Research question, hypothesis and scientific context

   The research question was: How does the height of the water level in a glass affect the frequency of the sound emitted by the glass when the side of the glass is hit with a spoon? The hypothesis predicted that as the water level in the glass increases, the frequency of the sound produced by the vibrating glass will decrease at an increasing, nonlinear rate. The processed data and the graph clearly show that frequency decreases as the height of the water level increases. The trendline that seems to best fit the data is a fourth order polynomial. Based on this, it seems that there is no obvious, clear mathematical relationship between the variables, although there might be a more complex connection. Results of similar experiments show graphs that have the same shape as the graph in this investigation (Lee 3, Nose 11). Moreover, in one of the experiments, the relationship between liquid height and frequency is modelled by a fourth order polynomial. In conclusion, the data and the results of other, similar experiments confirm that the hypothesis was correct.

II. Evaluation

1. Method

a. Strengths

The data collected in the experiment seems to be precise. First, we can see in Tables 1 and 2 that during the five trials carried out, the measured frequency values (except for one: 1030.2 Hz) were all within the absolute error of the average frequency. High precision is also confirmed by the negligible percentage uncertainty values in the measured frequencies (Table 2). In addition, the validity of the method and the reliability of the data are confirmed by the fact that both the general tendencies observed in the collected data and the shape of the graph are very similar to the outcome of experiments investigating the same topic.

b. Limitations, errors and possible improvements

Source and effects of error	Significance of error	Possible improvements
Inconsistency in the strength used when hitting the glass with the spoon: this could lead to a variation in the measured frequency values.	Low significance. Considering that the speed and the force we apply when we rub the rim of the glass with our fingers do not affect the frequency, we can assume that this is also true when hitting the glass with a spoon.	Create a simple system (possibly involving a simple pendulum) that can be used to hit the glass with a consistent force.
Inaccurate frequency measurement by the Google Science Journal App: this software has good opinions; however it is not considered a highly reliable measurement tool. Based on the data, measurements are precise, but might be lower or higher than the actual frequency values.	Medium significance. An upward or downward shift caused in all data points would still allow for a valid conclusion that answers the research question. The equation connecting the two variables would be affected.	Use two or three different ways to record the emitted frequency. These could include a spectrum analyser, a frequency counter or another App, for instance Decibel X.
Tape measure reading precision: it was not always easy to read the water level when using a measuring tape, which could have led to an upward or downward shift in individual data values.	High significance. If individual data values are shifted up or down (as opposed to the entire graph shifting - see previous error), this might lead to an incorrect trendline or no trendline at all.	Use a liquid level sensor to measure the height of the water level in the glass.

2. Extensions

   There are many ways that this experiment can be taken further. For example, glasses with different heights, volumes or shapes could be used. I included two possible, realistic extension ideas in the table below.

<table>
<tr><td colspan="3">Extension 1 research question:

How does the density of the liquid in a glass affect the frequency of the sound emitted by the glass when the side of the glass is hit with a spoon?</td></tr>
<tr><td>Independent variable</td><td>Dependent variable</td><td>Control variables</td></tr>
<tr><td>Density of liquid ($g\ cm^{-3}$)</td><td>Frequency of the emitted sound (Hz)</td><td>Type of glass

Height of liquid

Glass cross-sectional area

Air temperature</td></tr>
<tr><td colspan="3">Extension 2 research question:

How does the distance between a glass and a measuring device affect the intensity of the sound detected by the measuring device when the glass is hit by a spoon?</td></tr>
<tr><td>Independent variable</td><td>Dependent variable</td><td>Control variables</td></tr>
<tr><td>Distance between glass and measuring device (cm)</td><td>Intensity of the detected sound (dB)</td><td>Type of glass

Strength with which the glass is hit

Air temperature</td></tr>
</table>

Works Cited

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